# Thermodynamical Behavior in Parts of Closed Quantum Systems

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## Relaxation and Entanglement

**What about the following statement?**

**"Relaxation towards thermal equilibrium does not cause system-reservoir entanglement/correlations"**

Explicit and implicit opinions in the context of relaxation theories:

- It's not explicitly mentioned within the projection operator method. (Zwanzig, Nakajima, Kubo, etc.)
- A method based on iteration of the Liouville equation requires it. (Scully, Blum, Mahler, etc.)
- It underlies the dynamical semigroup approach. (Lindblad, Alicki, Lendi, etc.)
- It seems to be accepted in the case of weak interactions, even by scientists who doubt it in general (Pechukas, Weiss, Hänggi, etc.)

**Seems to be at least partially accepted.**

**How are correlations connected to a reduction of local purity?**

$$\hat{\rho}_c := \hat{\rho} - \hat{\rho}_s \otimes \hat{\rho}_r \qquad (\hat{\rho}_s := \mathrm{Tr}_r\{\hat{\rho}\}, \hat{\rho}_r := \mathrm{Tr}_s\{\hat{\rho}\})$$

Use the Bures metric as a measure of purity or "size" of an operator:

$$P_x^2 := \mathrm{Tr}\left\{\hat{\rho}_x^2\right\}$$

One can derive bounds for the "correlation/local" coefficient $\eta$:

$$\frac{P}{P_s P_r} - 1 \le \eta \le \frac{P}{P_s P_r} + 1 \qquad \eta := \frac{P_c}{P_s P_r}$$

If the full systems evolution is unitary and the initial state factorizes:

$$\frac{P_s(0)P_r(0)}{P_s(t)P_r(t)} - 1 \le \eta(t) \le \frac{P_s(0)P_r(0)}{P_s(t)P_r(t)} + 1$$

Even if the bath is assumed stationary:

$$\frac{P_s(0)}{P_s(t)} - 1 \le \eta(t) \le \frac{P_s(0)}{P_s(t)} + 1$$

So, *e.g.*, for a half decayed excitation in a 2-level system:

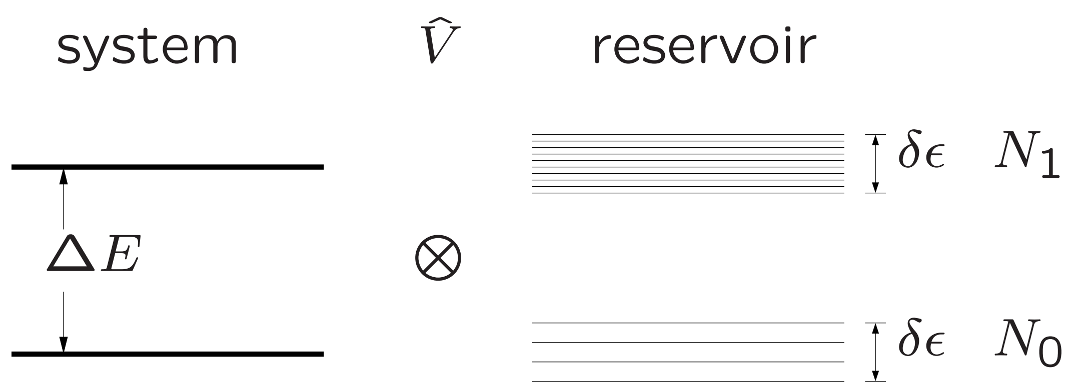
$$\sqrt{2} - 1 \le \eta(t) \le \sqrt{2} + 1$$

**Relaxation requires correlations!**

# Hilbertspace average method

**Can relaxation be described without assuming factorizability and/or a thermal reservoir state?**

Specific model of an unspecific situation:




$$\hat{H}_0 = \Delta E \hat{\sigma}^z + \frac{\delta\epsilon}{N_0}\sum_i^{N_0} i|i\rangle\langle i| + \frac{\delta\epsilon}{N_1}\sum_j^{N_1}(\Delta E + j)|j\rangle\langle j|$$

$$\hat{V} = \lambda \sum_{j,i}^{N_0 N_1} c_{ij}\hat{\sigma}^+|i\rangle\langle j| + c_{ij}^*\hat{\sigma}^-|j\rangle\langle i|$$

$c_{ij}$ : randomly distributed complex numbers

The Hilbertspace average method:

We describe the evolution of a pure state for a short time $\tau$ by a Dyson series up to 2. order:

$$|\Psi(t+\tau)\rangle \approx \hat{U}(t,\tau)|\Psi(t)\rangle$$

# Hilbertspace average method

For the excitation probability we get:

$$\langle \hat{P}_e(t+\tau)\rangle = \underbrace{\langle \Psi(t)|\hat{U}^{+}(t,\tau)\hat{P}_e\hat{U}(t,\tau)|\Psi(t)\rangle}_{:=\Pi(|\Psi\rangle,t,\tau)}$$

What did that help?

So far nothing. But now we compute the **Hilbertspace average** ($[\![...]\!]$) of $\Pi$ over all $|\Psi\rangle$, under the side condition $\langle \Psi|\hat{P}_e|\Psi\rangle = \langle \hat{P}_e(t)\rangle$.
Doing so we find

$$[\![\Pi(|\Psi\rangle,t,\tau)]\!] \approx \langle \hat{P}_e(t)\rangle - \gamma\tau\langle \hat{P}_e(t)\rangle$$

under some conditions for the model.

Can $\Pi$ be replaced by $[\![\Pi]\!]$ ? $\Rightarrow$ Yes, if the "$\Pi$-landscape" is flat!
It can be shown that the "flatness" of the $\Pi$-landscape increases with increasing number of levels of the environment, $N_0 + N_1 \Rightarrow$

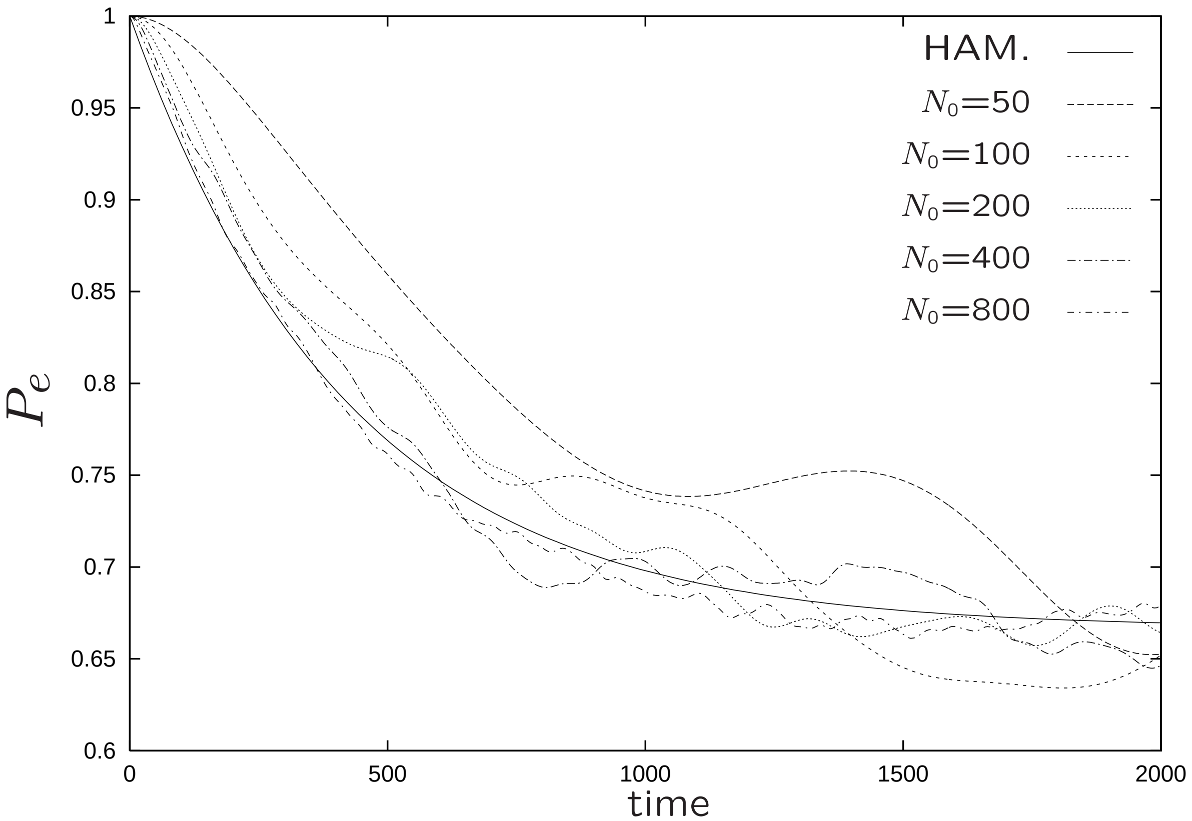
$$\langle \hat{P}_e(t+\tau)\rangle \approx \langle \hat{P}_e(t)\rangle - \gamma\tau\langle \hat{P}_e(t)\rangle$$

This is iterable, since it includes all possible states (entanglement, superpositions) and has no explicit time dependence. Thus for small $\tau$, $\Rightarrow$

$$\frac{d}{dt}P_e = -\gamma P_e$$

## Results

Solving the Schrödinger equation yields:




**- Maximum entanglement builds up during relaxation, independent of the interaction strength.**

**- The full system is always in a pure state, the bath is never in a thermal state.**

**- Nevertheless there is stochastic relaxation towards equilibrium.**

**- The Hilbertspace average method predicts this behavior consistently.**

More about this:


J. Gemmer, A. Otte, G. Mahler, Phys. Rev. Lett. **86**, 1927 (2001)

*Quantum Thermodynamics*, J. Gemmer, M. Michel, G. Mahler, LNP, Springer, hopefully out soon