Dynamics of Systems Coupled to Finite Markovian Environments: Deviations from Standard Quantum Master Equation Results

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Theory of Thermalizing Quantum Dynamics

Nakajima Zwanzig: projection operator technique, formally exact but practically unsolvable $\Rightarrow$

assume factorizing initial state, still unsolvable $\Rightarrow$

truncate at 2. order in the system-environment interaction strength $\Rightarrow$

$$\frac{\partial}{\partial t}\hat{\rho}_S = -\int_0^t \mathrm{Tr}_E \left\{ \left[\hat{I}(t), \left[\hat{I}(s), \hat{\rho}_S(s) \otimes \hat{\rho}_E(0) \right] \right] \right\} ds$$

higher order terms are hard to check.

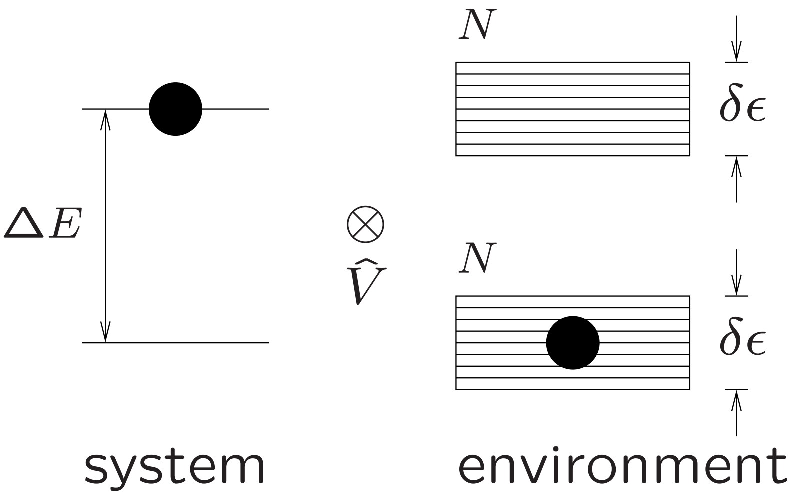
Markovian systems: system relaxation (to second order) much slower than decay of environmental autocorrelations $\Rightarrow$

easy to check $\Rightarrow$

$$\frac{\partial}{\partial t}\hat{\rho}_S(t) = \mathcal{L}\hat{\rho}_S(t)$$

Does good Markovicity justify the truncation of the Nakajima-Zwanzig equation at second order?

The Model:



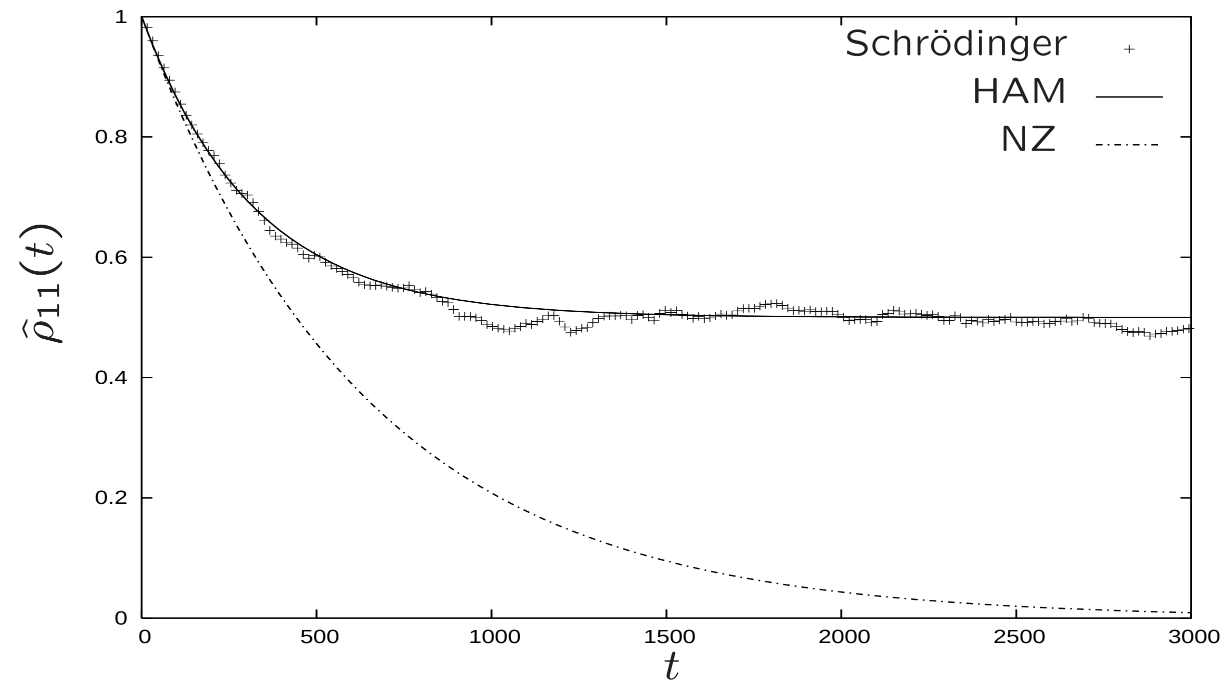
$$\hat{H}_0 = \Delta E \hat{\sigma}_z + \sum_n^N \frac{\delta\epsilon}{N} n |n\rangle\langle n| + \sum_m^N (\Delta E + \frac{\delta\epsilon}{N} m)|m\rangle\langle m|$$

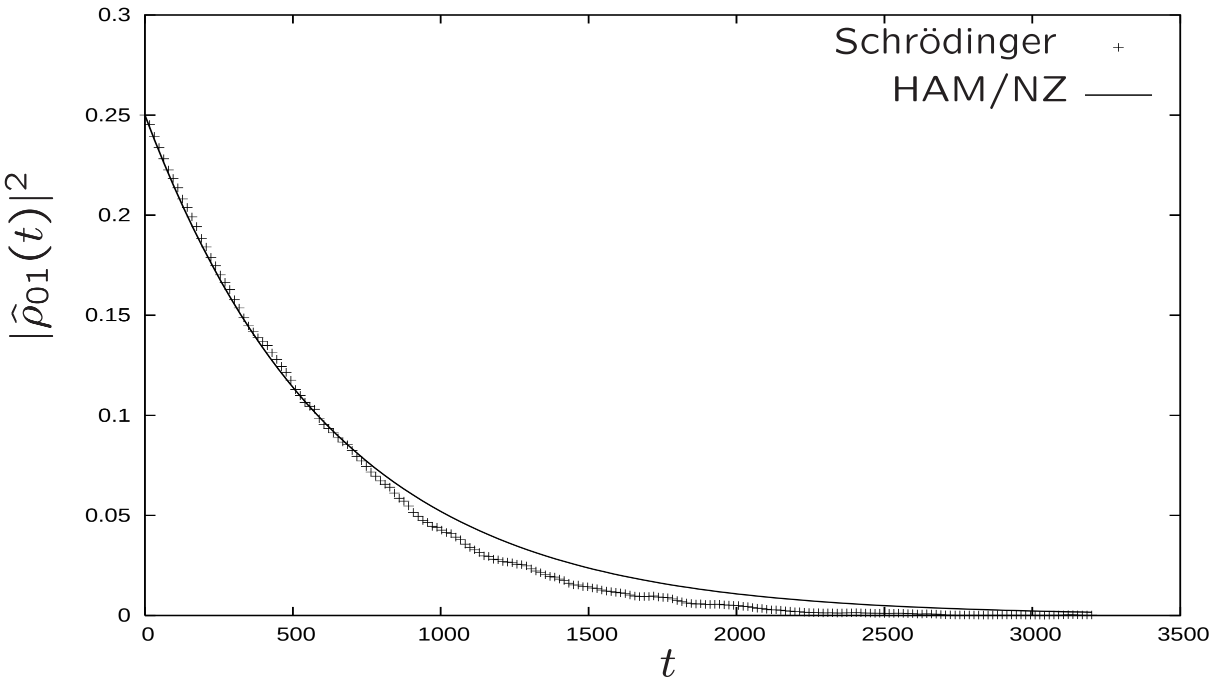
$$\hat{V} = \sum_{n,m} C_{nm} \hat{\sigma}^+ |n\rangle\langle m| + \hat{\sigma}^- |m\rangle\langle n|$$

- τ_{corr} depends on $1/\delta\epsilon$, τ_{relax} depends on $\bar{C}^2 N/\delta\epsilon$(?)
- C's: random complex numbers, $N = 500$, all parameters are chosen to yield $\tau_{relax}/\tau_{corr} \approx 100$ i.e., a Markovian system.
- we analyze pure (product) inital states

Will there be thermalization as predicted by the Nakajima-Zwanzig method?

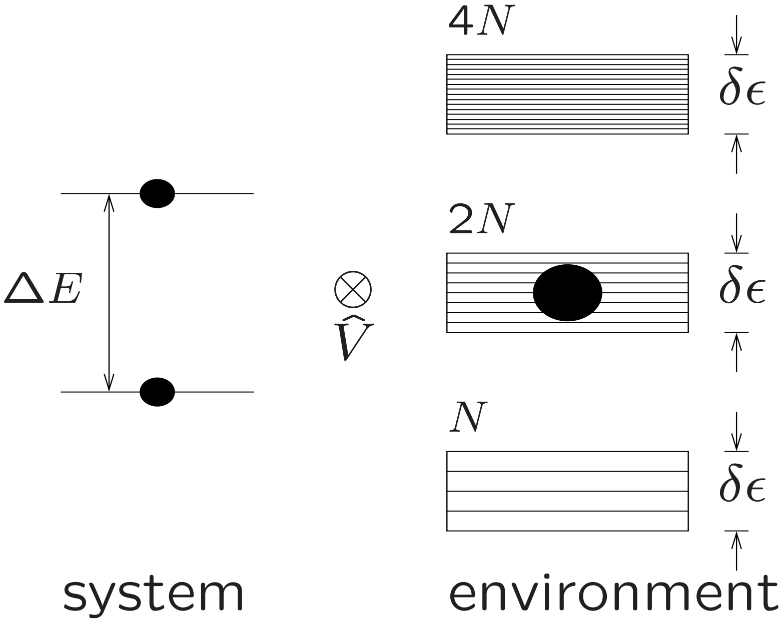
Thermalizing Dynamics

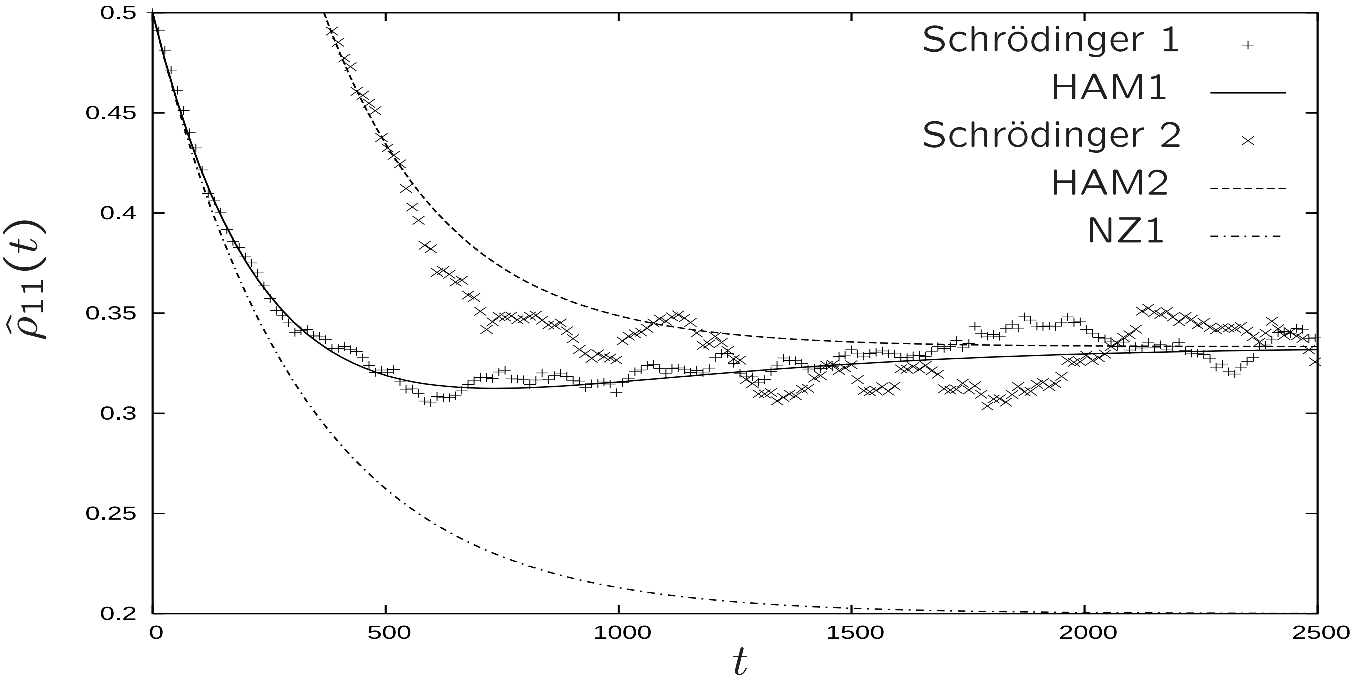




- NZ predicts $T = 0$, but system ends up at $T = \infty$
- equilibrium state does not depend on τ_{relax}/τ_{corr}
- Hilbert space Average Method (HAM) works!

Heat Bath Environment

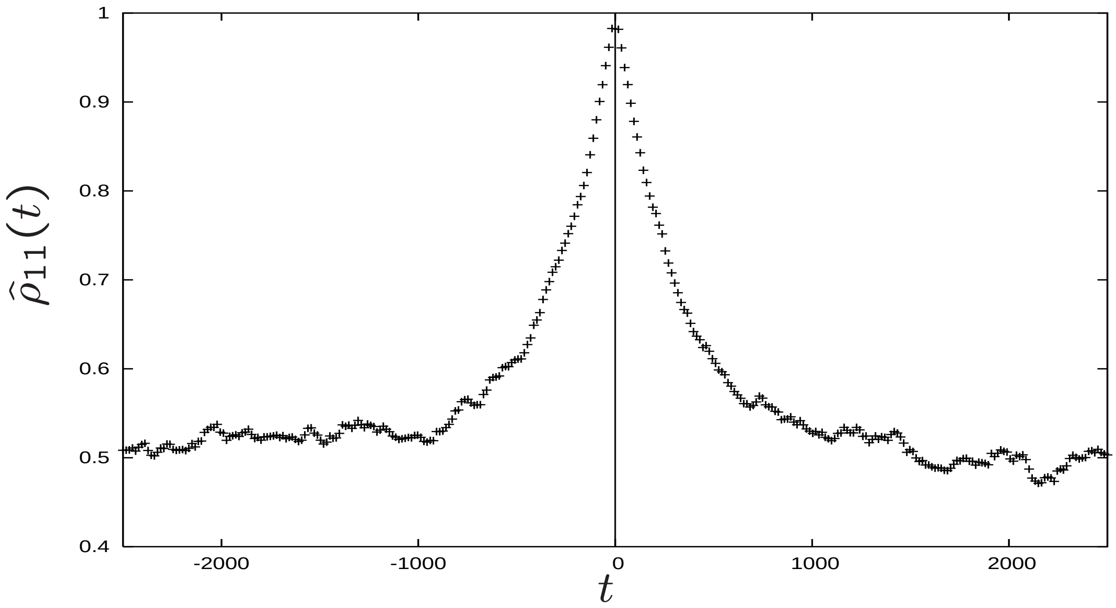




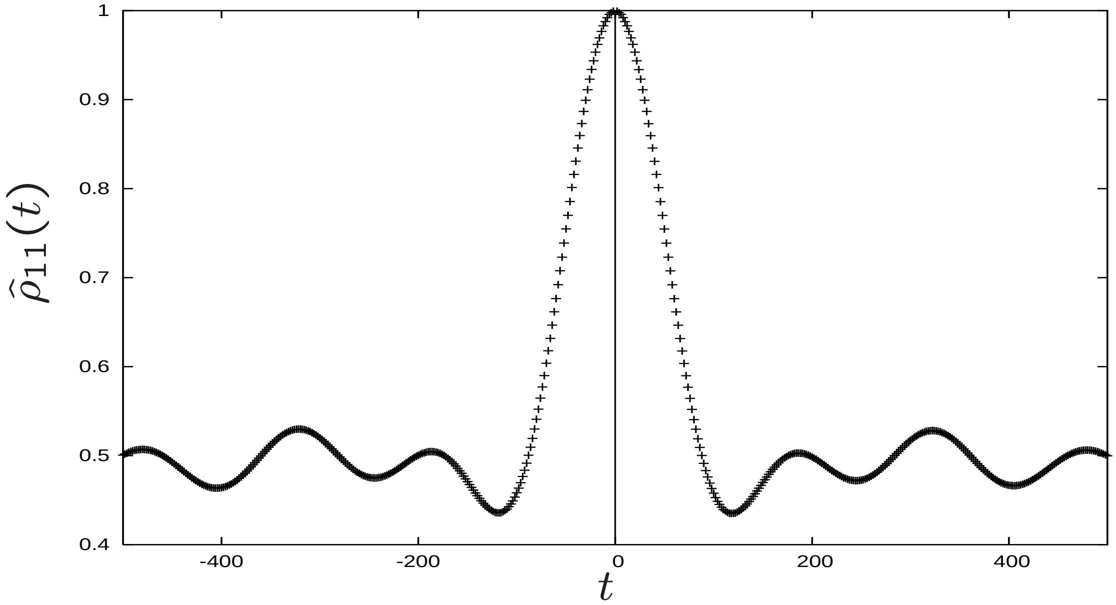
- equilibrium independent of initial state
- no Lindblad dynamics
- deviations from the equilibrium predicted by NZ

Time Reversal

Markovian:



non-Markovian:



- full dynamics are "time reversible" (no fixpoint)

- nevertheless: states that approach equilibrium are much more frequent than those departing from it.

The Bottom Line

Moderate size environments will most likely thermalize small quantum systems, although not necessarily in the way predicted by the Nakajima-Zwanzig technique

Detailed description of the underlaying concepts including HAM:

Quantum Thermodynamics
Emergence of Thermodynamic Behavior Within Composite Quantum Systems
Lecture Notes in Physics, Springer, Vol 657
J. Gemmer, M. Michel, G. Mahler (2004)
ISBN 3-540-22911-6

Less costly: Google ⇒ Quantenthermodynamik

(English site under construction.)

Thanks for paying attention at this time of day (night?) and enjoy Berlin!